

# Supplement to Paper : Interactive Visualization of Gaps and Overlaps for Large and Dynamic Sphere Packings

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Table 1 shows the rendering time and its breakdowns for overlaps/gaps under different phases, screen resolution and boundary conditions. In general, rendering overlaps under hard boundary conditions is faster than rendering under Periodic boundary conditions (PBCs) since there are more than  $N$  spheres to be drawn in the *set* step under PBCs. In this case, spheres that exceed the boundary need to be drawn on the opposite side as well. It is notable that rendering overlaps is usually slower on the late phase than on the start phase. Because spheres on late phases tend to avoid overlaps and the camera ray needs to travel further (i.e. examine more enter/leave points) to find the surface point of gaps or to see that there is no gap for a pixel (in this case, the ray has gone through all enter points for the pixel). That is vice versa for rendering gaps on late phases: it is likely that the camera ray stops on the first leave point.

Table 2 shows the precision of the volume calculation for a grid of spheres. No tiling is used here, so the computation gets inaccurate when the spheres become smaller than a pixel.

As shown in table 3, the memory requirements for the linked lists increase linearly with the viewport resolution and the summed cross sectional area of all spheres. The average length of a linked list is about proportional to the cubic root of the number of spheres of since the Collective Rearrangement (CR) algorithm tries to spread all spheres evenly in the container.

Screen resolution		512 x 512			768 x 768			1024 x 1024		
N	step	o	o(pbc)	g	o	o(pbc)	g	o	o(pbc)	g
1024*	set	0.19	0.31	0.31	0.4	0.75	0.74	0.61	1.19	1.18
1024*	draw	0.37	0.62	0.49	0.71	1.29	1.02	1.08	1.99	1.57
1024†	set	0.18	0.32	0.32	0.37	0.73	0.73	0.58	1.11	1.1
1024†	draw	0.35	0.68	0.48	0.68	1.39	1.01	1.12	2.08	1.53
50k*	set	0.47	0.85	0.83	1.02	1.53	1.49	1.68	2.44	2.37
50k*	draw	1.26	1.55	1.73	2.37	2.96	3.2	3.82	4.85	5.17
50k†	set	0.46	0.79	0.83	1.05	1.48	1.44	1.7	2.35	2.28
50k†	draw	1.34	2	1.54	2.58	3.76	2.98	4.11	6.14	4.78
10 <sup>6</sup> *	set	2	5.78	5.74	3.92	7.6	7.55	6.41	9.76	9.55
10 <sup>6</sup> *	draw	7.18	7.08	7.82	13.73	13.73	15.36	20.8	21.29	24.39
10 <sup>6</sup> †	set	1.85	6.19	6.02	3.66	7.93	7.87	5.92	9.92	9.79
10 <sup>6</sup> †	draw	8.02	10.67	6.94	15.51	20.51	13.54	22.83	30.84	20.79
10 <sup>7</sup> *	set	9.8	47.77	47.84	18.33	52.68	52.8	30.7	58.54	58.85
10 <sup>7</sup> *	draw	22.79	23.55	24.36	48.1	49.68	53.25	78.75	81.48	89.09
10 <sup>7</sup> †	set	9.45	49.45	49.39	17.75	54.33	54.2	29.86	60	59.89
10 <sup>7</sup> †	draw	31.89	30.45	20.43	64.8	63.27	43.38	107.08	101.87	71.01

**Table 1:** Rendering time in milliseconds. *o* stands for overlaps, *g* stands for gaps, *N* is the number of spheres. \* stands for the start phase, where all spheres are in their initial random positions. † stands for a late phase, where the volume of overlaps does not exceed 0.04. Step *set* is used to generate the linked lists and step *draw* sorts the linked list to find the appropriate depth for rendering overlaps/gaps. All measurements are conducted under perspective view where the front the side of the container matches the whole screen, i.e. all pixels have their own linked lists. Note that late phases are different for different screen resolutions, even with the same number of spheres *N*.

Radius	Absolute value (9 significant figures)			Relative error(%)	
	theoretical value	orth	pers	orth	pers
0.125	0.476401224	0.476380887	0.476392051	-0.004269	-0.001926
0.01	0.999731917	0.999734246	0.999729503	0.000233	-0.000242
0.001	0.999999732	0.999999258	0.999996914	-0.000047	-0.000282
0.125	0.523598776	0.523619113	0.523607949	0.003884	0.001752
0.01	0.000268083	0.000265754	0.000270497	-0.868603	0.900628
0.001	0.000000268	0.000000742	0.000003086	176.78	1051.138

**Table 2:** Volume calculation under screen resolution 768 x 768. All measurements are conducted under perspective view where the front the side of the container matches the whole screen. The cubic container is divided into 64 equal sized grids with a sphere on its center. All spheres have the same radius. The first 3 lines of results show the volume of gaps, the theoretical value, measured values and relative errors. The last 3 lines of results show the volume of spheres, the theoretical value, measured values and relative errors. **Orth** and **pers** stand for orthographic view and perspective view, respectively.

Screen resolution		512x512		768x768		1024x1024	
Boundary		Normal	PBCs	Normal	PBCs	Normal	PBCs
1024*	max	17	19	17	20	17	20
1024*	mean	3.3	5.8	3.3	5.8	3.3	5.8
1024*	mem	14	24.1	31.5	54.3	55.9	96.5
1024†	max	12	17	12	17	13	18
1024†	mean	3.1	5.3	3.1	5.2	3.1	5.4
1024†	mem	13.2	22.3	29.6	49.3	52.4	90.7
50k*	max	32	37	33	38	34	38
50k*	mean	7.7	9.5	7.6	9.5	7.6	9.5
50k*	mem	32.1	40	72.2	90	128.3	160.1
50k†	max	35	45	35	46	38	45
50k †	mean	7.7	9.3	7.7	9.4	7.7	9.3
50k †	mem	32.1	39	72.3	88.7	128.5	155.7
10 <sup>6</sup> *	max	59	61	56	56	58	57
10 <sup>6</sup> *	mean	15.2	16.8	15.2	16.8	15.2	16.8
10 <sup>6</sup> *	mem	63.6	70.6	143.1	158.9	254.5	282.5
10 <sup>6</sup> †	max	75	89	80	88	85	89
10 <sup>6</sup> †	mean	12.7	17	12.7	17	12.7	17
10 <sup>6</sup> †	mem	53.4	71.3	120.2	160.8	213.6	285.7
10 <sup>7</sup> *	max	92	91	94	94	96	99
10 <sup>7</sup> *	mean	28.2	30.1	28.2	30.1	28.2	30.1
10 <sup>7</sup> *	mem	118.3	126.3	266.2	284.4	473.3	505.5
10 <sup>7</sup> †	max	164	177	161	159	154	162
10 <sup>7</sup> †	mean	24.7	29.1	24.7	29.1	24.7	29.1
10 <sup>7</sup> †	mem	103.7	121.9	233.2	274.3	414.6	487.5

**Table 3:** Memory requirements: \* stands for the start phase, where all spheres are in their initial random positions. † stands for a late phase, where the volume of overlaps does not exceed 0.04. Max and mean are the maximal and mean length  $L$  of the linked lists in overlaps/gaps rendering algorithms among all pixels. Mem is the memory requirement for the linked lists in Mb (each entry needs two 32-bit machine words for the sphere depths, one 32-bit machine word for the sphere index, and one 32-bit machine word for the tail pointer). All measurements are conducted under perspective view where the front the side of the container matches the whole screen.